Math 141 **Test #3** **Fall 2004**

1. (40 pts) Let $f(x) = x^3 + 3x^2 - 9x - 27$. Use the first and second derivatives to determine

   (a) (10 pts) where f is increasing or decreasing, p. 181: 16

   (b) (10 pts) all relative extrema of f, if any, p. 181: 16

   (c) (10 pts) where f is concave upwards or concave downwards, p. 189: 11

   (d) (10 pts) all inflection points of f, if any. p. 189: 11

2. (10 pts) In the previous problem, sketch the graph of f, labeling all of its relevant features. p. 208: 29

3. (10 pts) Find the following limit. p. 199: 20

$$\lim_{x\to\infty} \frac{5x^3 + 2x + 8}{2x^3 + 4x^2 + 4}.$$

4. (10 pts) A rancher wishes to build two adjacent corrals, as shown in Figure 1. The two corrals are to have a combined area of 5000 ft^2. The total length of p. 216: 3

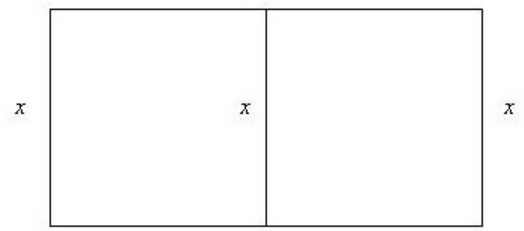

Figure 1: The rancher's corral

fence required is given by the function

$$f(x) = 3x + 2\left(\frac{5000}{x}\right).$$

Find the length and width of the corrals that will minimize the total length of fence required. Be sure to use either the first or the second derivative tests to verify your answer.

5. (10 pts) Write the iterative formula given by Newton's Method for the equation p. 226: 1

$$x^3 = 10,$$

then complete two iterations of the method, using a starting value of $x_1 = 2$.

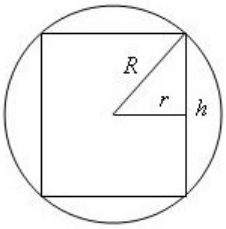

Figure 2: Cylinder inscribed in a sphere: side view

6. (10 pts) What is the volume of the largest right circular cylinder that can be inscribed in a sphere of radius R? Figure 2 shows a side view of the cylinder inscribed in the sphere. The volume of the cylinder is given by $V = \pi r^2 h$. I suggest that you express r in terms of h and substitute into the formula for the volume. Use the *second* derivative test to test the critical value(s). p. 216: 31

7. (10 pts) The distance from a point to a tree is measured to be 100 ft. From that point, the angle from the horizontal to the top of the tree is measured. The formula p. 234: 35, 43

$$h = b \tan \theta$$

is used to calculate the height of the tree, where b is the distance to the tree, θ is the angle, and h is the height of the tree. The angle is found to be 30°, with a possible error of 1°. Use differentials to approximate the maximum possible error in the calculated height of the tree.